**Cooper Annualization Research**

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1. Basic annualization of returns:

Annualize a stream of returns :

This can be applied to a single return of any duration, or multiple returns of any duration.

Our assumptions are as follows: we first invest a non-zero amount of money, and then from then on, we can only lose 100% of what we have. This implies, respectively, that and are both finite, and that for all .

2. Annualization of arithmetic differences of returns

Given a stream of , and

Given a stream of ,

3. To answer this, one needs to understand annualization ().

But is a type of averaging. Thus, one needs to understand averaging.

4. General definition of an average:

a) Arithmetic average

b) Geometric average

c) What is the generalization of this process?

, where

i) is the function, of the list , that is to be preserved,

ii) might be required to fulfill some restrictions, like:

* symmetry
* monotonicity

iii) is the solution to:

d) Note:

i) One solution for any list of identical values,

for any function, , the solution is the identical value:

is a solution.

ii) The function must be written as an explicit function of the set of

terms in the list that one wishes to average, so that the substitution process is well defined.

5. Numerical examples (Arithmetic, Geometric and Annualization)

a) :

b)

c) :

6. Annualization of an active return

a) Naive approach

Example:

However,

But from 4.d.i) this should equal 50%, not 51.8%.

So

b) Applying general definition of an average to the annualization of an active return .

Preserve the difference of the total returns:

To find the average of , write as: .

Thus, applying the general definition of average we get:

Applying this result to the numerical example in 6.a), we get:

As required, we see that 0.5 = 50% is a solution for , the annualization of the active return.

7. The first step is getting some code to work that will solve for in

for the general case of sets of and for , and where return factors and are all yearly. (Even when there are numerous years and where and tend to track each other closely.)

8. The second step is to find and then to get some code to work that will solve the proper generalization of

for in the general case of sets of and for , and where the return factors and are for any combinations of durations.

An intermediate step would be to get the generalization of this equation to work for ~250 days (i.e. all input durations are equal to a day), where the we would seek is the annualized active return for the whole year.

Note:

1. Numerically,
2. Since is not a return factor, the equation is not
3. Extracting roots or taking logarithms (in order to, say, simplify the above equation) result in the loss of solutions, which is undesirable.

9. The next “step” is to generalize this process (if and to the extent possible) so that it applies not to just the arithmetic differences of returns, , but to deterministic ex-post functions like

Tracking Error:

Information Ratio:

I published parts of the initial remarks above and some ill-formed suggestions for approaching some of the more challenging issues involved in these steps as a chapter in the book “Advanced Portfolio Attribution Analysis” edited by Carl Bacon. But as far as I know, no one has ever seriously addressed either of the last two steps.

10. Aside: consider the following two annualizations:

a)

b)

What is the difference between the two annualizations?

1. gives the average contribution of a year to
2. gives an that is the average over the period that preserves